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Cosmic acceleration and crossing of $w = -1$ barrier in non-local Cubic Superstring Field Theory model

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Abstract: We show that the rolling of the Cubic Superstring Field Theory (CSSFT) non-local tachyon in the FRW Universe leads to a cosmic acceleration with a periodic crossing of the $w = -1$ barrier at large time. An asymptotic solution for the tachyon field and Hubble parameter is constructed explicitly linearizing non-local equations of motion. For a small Hubble parameter a period of oscillations is a number entirely determined by parameters of the CSSFT action.

Keywords: String Field Theory, Cosmology of Theories beyond the SM.

Contents

1. Introduction

The combined analysis of the data on type Ia supernovae, galaxy clusters measurements and WMAP (Wilkinson Microwave Anisotropy Probe) gives clear evidence of the accelerated expansion of the universe [1]–[5]. The cosmological acceleration suggests that the current Universe is dominated by the smoothly distributed, slowly varying Dark Energy (DE). Recent results of WMAP [6] together with the data on Ia supernovae strongly support that the DE state parameter is currently close to -1:

$$w = -0.97^{+0.07}_{-0.09}$$

or without an a priori assumption that the Universe is flat and together with the data on large-scale structure and supernovae $w = -1.06^{+0.13}_{-0.08}$.

From the theoretical point of view the above mentioned domain of w covers three essentially different cases. The first case, $w > -1$, is achieved in the quintessence models [7, 8] containing an extra light scalar field which is not in the Standard Model set of fields [9]. The second case, $w = -1$, is the cosmological constant [10, 11]. The third case, $w < -1$, is called the "phantom" case and can be realized using a scalar field with a ghost (phantom) kinetic term. In this case all natural energy conditions are violated and there are problems of instability at the classical and quantum levels [12, 13].

Since experimental data do not contradict a possibility that $w < -1$ and moreover the direct search strategy to test inequality $w < -1$ has been proposed [14] the study of such models attracts a lot of attention. Some projects [15] explore whether w varies with the time or an exact constant. Varying w obviously corresponds to a dynamical model of the DE (see [16] for a review) which generally speaking includes a scalar field.[1]

[1] Modified models of GR also generate an effective scalar field (see for example [17] and refs. therein).

A possible way to avoid the instability problem for models with $w < -1$ is to consider the phantom model as an effective one, arising from a more fundamental theory without a negative kinetic term. In this paper we develop in more details a cosmological SFT tachyon model [18]. The model is based on the SFT formulation of the fermionic NSR string with the GSO$-$ sector [19]. In this model a scalar field is the open string tachyon, which describes according to the Sen's conjecture [20] (see [21] for a review) a dynamical transition of a non-BPS D-brane[2] to a stable vacuum. Since our model is an approximation to the string theory all stability issues are related to a stability of a VSFT (a Vacuum String Field Theory, i.e. SFT in a true vacuum) and one has to discuss only an application of our approximation to a full string theory. There are general arguments [23] that there does not exist a local scalar field model for a phantom Universe without an UV pathology. In a recent paper [24] it has been proposed a phantom model without UV pathology in which a vector field is used.

The scalar model we investigate in this paper is a nonlocal one. Our goal is to construct analytic solutions to linearized Friedmann equations at large times. A characteristic feature of this model in the flat background is a presence of a rolling tachyon solution [25 – 27]. This property force us to consider a fermionic string instead of bosonic one where such a solution does not exist [28 – 30].[3] However one might expect an existence of a rolling solution in bosonic strings in a non-flat space. The dynamics of a non-local tachyon on a cosmological background in the Hamilton-Jacobi formalism is studied in [32]. The rolling tachyon solutions in boundary SFT have been considered in [33] and a crossing of the possitive/negative barrier for pressure has been found.

We find that during the evolution in the FRW Universe the SFT tachyon goes to its minimum oscillating with an exponentially decreasing amplitude at large time. This is similar to the flat case. Consequently the Hubble parameter goes to a constant, and the state and deceleration parameters go to -1 all oscillating around their asymptotic values. The DE state parameter w crosses the phantom divide $w = -1$ during the evolution.

Models with a crossing of the $w = -1$ barrier are also a subject of recent studies. Simplest models include two scalar fields (one phantom and one usual field, see [34 – 36] and refs. therein). General κ-essence models [37] can have both $w < -1$ and $w \geqslant -1$ but a dynamical transition from the region $w \geqslant -1$ to the region $w < -1$ or vice versa is forbidden under general assumptions [38] and is possible only under special conditions [39].

In our case a non-locality provides a crossing of the $w = -1$ barrier in spite of the presence of only one scalar field. The Universe driven by the non-local scalar field exhibits an acceleration but because of oscillations quintessence and phantom phases change one each other with time.

The paper is organized as follows. In section 2 we setup the cosmological model which is an approximation of the CSSFT describing a non-BPS brane within the level truncation scheme in the FRW Universe. In section 3 we present details of the tachyon dynamics in the flat space at large times where an approximation linear in fluctuations

[2]DE models based on brane-world scenarios are presented in [22].

[3]For another approach using a Dirac-Born-Infeld action to describe the tachyon dynamics, see [31] and refs. therein

around a non-perturbative vacuum is valid. We compare this linear approximation with a numeric solution to full equations. In section 4 we study the tachyon dynamics in the FRW background again using a linear approximation to the Friedmann equations and find out that the obtained solution describes an accelerating Universe. In section 5 we discuss cosmological consequences of the obtained results and point out further directions of studying this type of models.

2. Setup

An action for the tachyon in the CSSFT [40, 41] in the flat background[4] when fields up to zero mass are taken into account is found to be [19, 25]

$$S_{\mathrm{SFT}}=\frac{1}{g_o^2\alpha'^2}\int dx\left(u^2(x)-\frac{\alpha'}{2}\eta^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x)+\frac{1}{4}\phi^2(x)+\frac{e^{2\lambda}}{3}\tilde{\phi}^2(x)\tilde{u}(x)\right)$$

where $\phi(x)$ is the tachyon field, $u(x)$ is an auxiliary field,

$$\tilde{\phi}=e^{\alpha'\lambda\Box}\phi,$$

and $\lambda=-\log\frac{4}{3\sqrt{3}}\approx 0.2616$. η is the flat Minkowskian metric, $\Box=\eta^{\mu\nu}\partial_\mu\partial_\nu$. For simplicity we will use hereafter $\alpha'=1$ units in which all fields, coordinates and the coupling constant g_o are dimensionless.

An auxiliary field $u(x)$ can be integrated out to yield

$$S_{\mathrm{tach}}=\frac{1}{g_o^2}\int dx\left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x)+\frac{1}{4}\phi^2(x)-\frac{e^{4\lambda}}{36}\left(\widetilde{\tilde{\phi}^{\,2}(x)}\right)^2(x)\right).$$

An approximation[5] that u has no the tilde simplifies the last term in this action. Namely, under this assumption and a rescaling $x\to 2\sqrt{2\lambda}x$, $\phi\to\frac{3}{\sqrt{2}}e^{-2\lambda}\phi$, and $g_o\to 12\lambda e^{-2\lambda}g_o$ the action for the tachyon becomes

$$S_{\mathrm{tach,\ approx}}=\frac{1}{g_o^2}\int dx\left(-\frac{\xi^2}{2}\eta^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x)+\frac{1}{2}\phi^2(x)-\frac{1}{4}(e^{\frac{1}{8}\Box}\phi)^4(x)\right)\tag{2.1}$$

where $\xi^2=\frac{1}{4\lambda}\approx 0.9556$. The last term in this action contains an infinite number of derivatives. Just due to this nonlocal factor a novel behavior in a dynamics of the tachyon field appears [25].

We continue with a cosmological scenario where our Universe is considered as a D3-brane embedded in 10-dimensional space-time following the lines of [18]. To this end we minimally couple the tachyon to Einstein gravity by covariantizing the above action (2.1). Though everyone understands this is not a full theory it is obvious that such a minimal

[4] We always use the signature $(-,+,+,+,\dots)$.

[5] As it was shown in [25, 26] this approximation can be applied to study a special class of rolling solutions and related questions. Further comments are put in the appendix.

coupling gives a starting point to have an insight into the problem of string modes behavior in curved space-time. Therefore, the action becomes

$$S = \int dx\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{1}{g_o^2}\left(-\frac{\xi^2}{2}g^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x) + \frac{1}{2}\phi^2(x) - \frac{1}{4}\Phi^4(x) - \Lambda\right)\right) \quad (2.2)$$

where

$$\Phi = e^{\frac{1}{8}\Box_g}\phi, \quad \Box_g = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu.$$

Here g is the metric, κ is a gravitational coupling constant and we choose such units that it is dimensionless, Λ is a constant.

In the present analysis we focus on the four dimensional Universe with the spatially flat FRW metric which can be written as

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2)$$

with $a = a(t)$ being a space homogeneous scale factor. In this particular case $\Box$ is expressed through a as

$$\Box_g = -\partial_t^2 - 3H\partial_t + \frac{1}{a^2}\partial_{x_i}^2$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and the dot denotes the time derivative.

The Friedmann equations for the space homogeneous tachyon field have the form [18]

$$3H^2 = \frac{\kappa^2}{g_o^2}\left(\frac{\xi^2}{2}\dot{\phi}^2 - \frac{1}{2}\phi^2 + \frac{1}{4}\Phi^4 + \mathcal{E}_1 + \mathcal{E}_2 + \Lambda\right), \quad (2.3a)$$

$$\dot{H} = \frac{\kappa^2}{g_o^2}\left(-\frac{\xi^2}{2}\dot{\phi}^2 - \mathcal{E}_2\right) \quad (2.3b)$$

where

$$\mathcal{E}_1 = -\frac{1}{8}\int_0^1 ds(e^{\frac{1}{8}s\mathcal{D}}\Phi^3)\mathcal{D}e^{-\frac{1}{8}s\mathcal{D}}\Phi, \quad \mathcal{E}_2 = -\frac{1}{8}\int_0^1 ds(\partial_t e^{\frac{1}{8}s\mathcal{D}}\Phi^3)\partial_t e^{-\frac{1}{8}s\mathcal{D}}\Phi$$

with

$$\Phi = e^{\frac{1}{8}\mathcal{D}}\phi, \quad \mathcal{D} = -\partial_t^2 - 3H(t)\partial_t.$$

The equation of motion for the tachyon is

$$\left(\xi^2\mathcal{D} + 1\right)e^{-\frac{1}{4}\mathcal{D}}\Phi = \Phi^3. \quad (2.4)$$

The latter equation is in fact the continuity equation for the cosmic fluid. To see this explicitly the following operator relation is useful

$$\lambda\int_0^1 ds\left((e^{s\lambda\Box_g}\Box_g\phi)Pe^{(1-s)\lambda\Box_g}\psi - (e^{s\lambda\Box_g}\phi)\Box_g Pe^{(1-s)\lambda\Box_g}\psi\right) =$$

$$= (e^{\lambda\Box_g}\phi)P\psi - \phi Pe^{\lambda\Box_g}\psi - \lambda\int_0^1 ds(e^{s\lambda\Box_g}\phi)\pi e^{(1-s)\lambda\Box_g}\psi$$

where P and π are operators satisfying $[\Box_g, P] = \pi$.

Equations (2.3) are complicated because of the presence of an infinite number of derivatives and a non-flat metric. Before we are going to present a construction of an asymptotic solution to these equations to gain an insight into the problem of dealing with an infinite number of derivatives we shortly review the results of an analysis of action (2.1) in the flat space [25–27].

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3. The rolling tachyon in the flat background at large time

Approximate action for the tachyon (2.1) in the flat background was already studied in [25–27][6] and we will present the results relevant for the sequel. The equation of motion for space homogeneous configurations of the tachyon field is found to be

$$\left(-\xi^2\partial_t^2+1\right)e^{\frac{1}{4}\partial_t^2}\Phi(t)=\Phi(t)^3. \tag{3.1}$$

where $\Phi=e^{-\frac{1}{8}\partial_t^2}\phi$. The operator $e^{s\partial_t^2}$ for positive s can be rewritten in an integral form and this allows further to implement an iteration procedure which is capable to find a numeric solution to the problem.

If we are interesting in the rolling tachyon solution then we have in mind the following picture. The tachyon field starts from the origin (may be with a non-zero velocity), rolls down to the minimum of the tachyon potential[7] and eventually stops in the minimum. In our notation the minima are located at $\Phi_0=\pm 1$. For $\xi^2\neq 0$ and $\xi^2<\xi_{\text{cr}}\approx 1.38$ there are damping fluctuations near the minimum [26]. Let us note that in our case $\xi^2<\xi^2_{\text{cr}}$. To analyze the behavior at large time one can linearize equation (3.1) as $\Phi=\Phi_0-\delta\Phi$ keeping only liner in $\delta\Phi$ terms. A substitution yields the following equation for $\delta\Phi$

$$\left(-\xi^2\partial_t^2+1\right)e^{\frac{1}{4}\partial_t^2}\delta\Phi=3\delta\Phi. \tag{3.2}$$

At this point we want to mention two facts about this equation. First, if one omits the non-local exponential operator then one arrives to an equation for the harmonic oscillator

$$\left(\xi^2\partial_t^2+2\right)\delta\Phi=0$$

which does not give the desired behavior at large time. Second, if we expand the non-local operator and keep only the constant and second derivative terms the situation depends on the value of ξ^2 and for $\xi^2>\frac{1}{4}$ (which is our case) remains qualitatively the same. Using that $e^{\partial_t^2}e^{-mt}=e^{m^2}e^{-mt}$ one can try to find a solution to equation (3.2) of the form $\delta\Phi(t)=Ce^{-mt}$ where for m we have the following characteristic equation

$$\left(-\xi^2m^2+1\right)e^{\frac{1}{4}m^2}=3. \tag{3.3}$$

This is exactly equation (3.27) in [18] with a redefinition $m_{\text{our}}=im_{[18]}$.

Note, that this solution is valid only at large time. One expects thereby that the non-local operator plays a crucial role in the asymptotic regime. The latter characteristic equation for m is solved in appendix A. Here only the results are used. A general solution for $\delta\Phi$ is an infinite sum

$$\delta\Phi(t)=\sum_k(A_ke^{-m_kt}+B_ke^{m_kt})$$

because infinitely many roots m_k^2 exist. This reflects the presence of an infinite number of derivatives. The latter means an infinite number of initial conditions which however are not

[6] See also [28, 29] for a cubic action.

[7] The potential is equivalent in terms of both ϕ and Φ since $\phi=\Phi$ for zero momentum.

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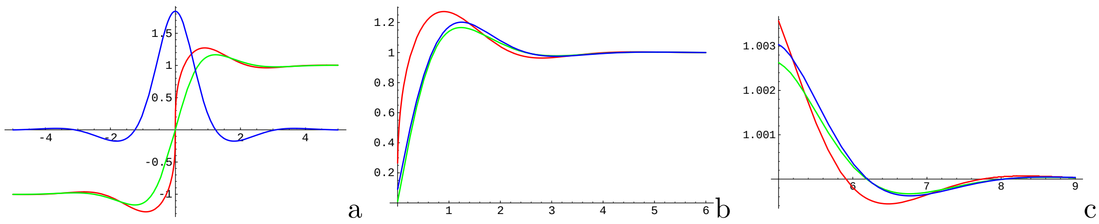

Figure 1: a. Solution to equation (3.1): Φ (red line), ϕ (green line), $\dot{\phi}$ (blue line) for $\xi^2 = 0.9556$; b. An approximation to this solution $1 - \delta\phi$ given by (3.5) (blue line); c. The same plots as in b in a different scale.

arbitrary (see [28] for a discussion on this point). Moreover in the full an action asymptotic behavior is subject to initial conditions in the origin. In other words considering the full non-linear equation and imposing initial conditions in the beginning of the evolution one arrives to a specific asymptotic configuration. We assume in our case a solution converging to an asymptotic value exists. For the definition of m_k given in appendix a vanishing condition imposes $B_k = 0$ for all k and a reality condition for $\delta\Phi$ is formulated as $A_k = A^*_{-1-k}$. Therefore, the most general real vanishing solution to equation (3.2) is

$$\delta\Phi(t) = \sum_{k\geq 0}(A_k e^{-m_k t} + A_k^* e^{-m_k^* t}). \tag{3.4}$$

Hence one yields a real function of time which oscillates with an exponentially decreasing amplitude. The function $\Phi(t)$ in turn goes to 1 oscillating with the decreasing amplitude near the asymptotic value. Recall that the solution constructed is valid only at large time. Let us also note that in a UV region, $k \geq 1$, one has to take into account string modes next to leading ones and this is beyond of our approximation.

The main (i.e. the most slowly vanishing) contribution in (3.4) is given by $k = 0$ and can be represented as

$$\delta\phi(t) = Ce^{-rt}\sin(\nu t + \varphi) \quad \text{where} \quad r \approx 1.1365, \; \nu \approx 1.7051 \tag{3.5}$$

where we passed to $\delta\phi = e^{\frac{1}{8}\partial_t^2}\delta\Phi$. All other k will give a faster convergence and will play a role of small corrections. We find that a rather simple choice of constants $C \approx 1$ and $\varphi \approx 2$ in (3.5) matches the rolling solution presented in figure 1a. Figures figure 1b and figure 1c contain corresponding plots.

Note that for $\xi^2 = 0$ (this case corresponds to a p-adic string [42, 43] with $p = 3$) equation (3.2) is simplified drastically and we have

$$m_k^2 = 4(\log 3 + 2\pi k i)$$

where again different branches may be considered. The principal branch is $k = 0$ and it corresponds to the rolling solution [25 – 27]. Other k do not match the rolling solution. They will give oscillations which were not present in a numeric analysis. Absence of such an oscillating behavior is related to initial conditions in the origin. One can also say that m_k for $k = 1, 2, \ldots$ are in a UV region beyond of our approximation.

The main conclusion of this section is that the non-local operator $e^{s\partial_t^2}$ for positive s acts like a friction. For $\xi^2 = 0$ the late time behavior of the tachyon is just a smooth rolling with a monotonically vanishing velocity to the minimum of the potential. This is not surprising mathematically but looks rather strange physically. Term with ξ^2 in the l.h.s. of equation (3.2) accelerates the tachyon but for actual ξ^2 a friction becomes stronger and the tachyon eventually stops in the minimum.

Another point of view is that for the considered ξ^2 time intervals when the kinetic energy on space homogeneous configurations is negatively defined dominate providing a phantom behavior for the tachyon field. This justifies a phantom approximation used in [44].

4. The rolling tachyon in the FRW Universe at large time

We consider action (2.2) for the tachyon coupled to the gravity. We are going to employ an analog of the asymptotic expansion used above in section 3. To this end we have to expand $\phi = \phi_0 - \delta\phi$ (and accordingly $\Phi = \phi_0 - \delta\Phi$, where $\delta\Phi = e^{\frac{1}{8}\mathcal{D}}\delta\phi$) in Friedmann equations (2.3). In our notations $\phi_0 = \pm 1$ and the resulting equations read

$$3H^2 = \frac{\kappa^2}{g_o^2}\left(\frac{\xi^2}{2}\dot{\delta\phi}^2 - \frac{1}{2}\delta\phi^2 + \frac{3}{2}\delta\Phi^2 + \delta\mathcal{E}_1 + \delta\mathcal{E}_2 + \Lambda_0\right), \tag{4.1a}$$

$$\dot{H} = \frac{\kappa^2}{g_o^2}\left(-\frac{\xi^2}{2}\dot{\delta\phi}^2 - \delta\mathcal{E}_2\right) \tag{4.1b}$$

where

$$\delta\mathcal{E}_1 = -\frac{3}{8}\int_0^1 ds e^{\frac{1}{8}s\mathcal{D}}\delta\Phi\mathcal{D}e^{-\frac{1}{8}s\mathcal{D}}\delta\Phi, \quad \delta\mathcal{E}_2 = -\frac{3}{8}\int_0^1 ds\partial_t e^{\frac{1}{8}s\mathcal{D}}\delta\Phi\partial_t e^{-\frac{1}{8}s\mathcal{D}}\delta\Phi$$

and $\Lambda_0 = -\frac{1}{4} + \Lambda$. The equation of motion for the tachyon field ϕ (2.4) after expansion has the form

$$(\xi^2\mathcal{D} + 1)e^{-\frac{1}{4}\mathcal{D}}\delta\Phi = 3\delta\Phi. \tag{4.2}$$

At this point we assume that in this approximation only the constant term H_0 in an expansion of H survives in the latter equation. A validity of this assumption will become clear after a solution will be constructed. A value of H_0 can be determined from equation (4.1a). However, even in this case an analytic solution to equation (4.2) in a closed form is not achievable so far. Instead of this we are looking for a solution constructed from eigenfunctions of the operator $\mathcal{D}_0 = -\partial_t^2 - 3H_0\partial_t$. They have the following form

$$\delta\Phi = Ae^{-\bar{m}_+ t} + Be^{-\bar{m}_- t} \quad \text{where} \quad \bar{m}_\pm = \frac{3}{2}H_0 \pm \sqrt{\frac{9H_0^2}{4} + m^2}.$$

To become a solution to equation (4.2) with a constant $H = H_0$ a parameter m should be determined by means of transcendental equation (3.3) already appeared in the flat case. A general solution to equation (4.2) is

$$\delta\Phi = \sum_k \left(A_k e^{-\bar{m}_{+k}t} + B_k e^{-\bar{m}_{-k}t}\right).$$

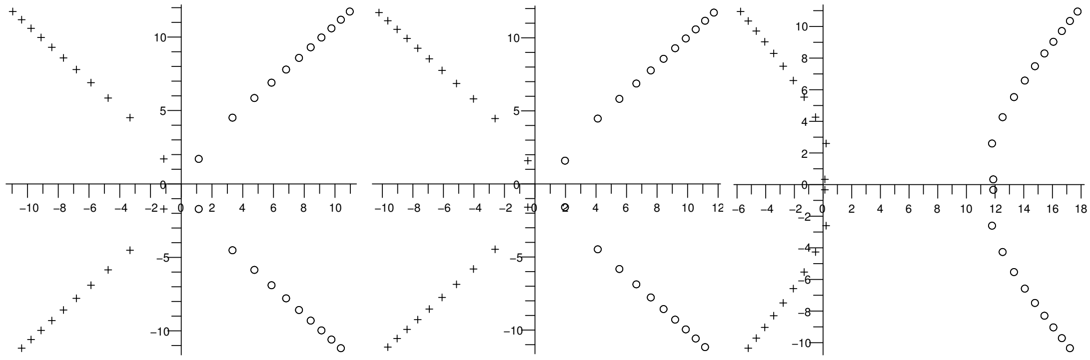

Figure 2: $\bar{m}_{\pm k}$ for $H_0 = 0$ (left), $H_0 = \frac{1}{2}$ (middle), and $H_0 = 4$ (right). Circles correspond to the plus and crosses to the minus sign.

An analysis of this solution goes in an analogy with the flat case considered in section 3.

For $\bar{m}_{\pm k}$ we have

$$\bar{m}_{\pm k} = \bar{r}_{\pm k} + i\bar{\nu}_k,\ \bar{r}_{\pm k} = \frac{3}{2}H_0 \pm \frac{|\beta_k|}{\bar{v}},\ \bar{\nu}_k = \mathrm{sign}(\beta_k)\frac{\bar{v}}{2},$$
$$\bar{v} = \sqrt{-2\left(\frac{9H_0^2}{4} + \alpha_k\right) + 2\sqrt{\left(\frac{9H_0^2}{4} + \alpha_k\right)^2 + \beta_k^2}}$$

where a notation $m_k^2 = \alpha_k + i\beta_k$ is used. However, in contrary with the flat case a selection of vanishing branches is not so obvious. It depends on a particular value of H_0. Notice, that $\bar{\nu}_k$ is always non-zero, i.e. a solution is oscillating for any k. Fortunately, the symmetry $\bar{m}_{\pm k} = \bar{m}^*_{\pm(-1-k)}$ persists. Thus a reality condition for $\delta\Phi$ is $A_k = A^*_{-1-k}$ and $B_k = B^*_{-1-k}$, and the most general real vanishing solution becomes

$$\delta\Phi = \sum_{k\geq 0}{}' \left(A_k e^{-\bar{m}_{+k}t} + A_k^* e^{-\bar{m}^*_{+k}t}\right) + \sum_{k\geq 0}{}' \left(B_k e^{-\bar{m}_{-k}t} + B_k^* e^{-\bar{m}^*_{-k}t}\right) \tag{4.3}$$

where the prime means that only k for which the behavior is vanishing should be taken into account.

Let us spend few lines discussing a question about the main (i.e. most slowly vanishing) contribution. A parameter H_0 complicates the story since $H_0 > 0$. Thus $k = 0$ is not necessary the main contribution. Moreover, for specific values of H_0 and k both plus and minus in $\bar{r}_k$ may give a positive number. In this case a corresponding B_k term is allowed and this is new compared to the flat case. To illustrate the situation $\bar{m}_k$ are drawn in figure 2 for different values of H_0. On these plots each point corresponds to a specific k. For $k \geq 0$ the imaginary part is positive and grows with k. For $k \leq -1$ the imaginary part is negative and grows in absolute value with $|k|$. Circles correspond to the plus and crosses to the minus sign in $\bar{m}_{\pm k}$. Points with smallest positive real parts form the main contribution and one sees that a selection of such points is not so transparent. However, for the sequel we will need only to state that one can make such a selection. Thus for the main contribution in (4.3) we have

$$\delta\phi = Ce^{-\bar{m}t} + C^* e^{-\bar{m}^*t} \tag{4.4}$$

where we have passed to $\delta\phi = e^{-\frac{1}{8}\mathcal{D}}\delta\Phi$ and an index $\pm k$ in $m_{\pm k}$ is omitted since a specific choice of k and a branch of the square root is made. Also for some values of H_0 it is possible that two close points, say k and $k+1$ for $k>0$ will have equal and smallest positive real parts. In this case they will only differ by a frequency of oscillations which will be suppressed with an equal exponential factor. However, for simplicity we will not consider this situation. Substituting solution (4.4) into Friedmann equation (4.1b) one yields after an integration using relation (3.3)

$$H = H_0 + \frac{\kappa^2}{g_o^2}\left(\frac{C^2\bar{m}}{16}e^{-2\bar{m}t}\left(1+4\xi^2-\xi^2m^2\right) + \frac{C^{*2}\bar{m}^*}{16}e^{-2\bar{m}^*t}\left(1+4\xi^2-\xi^2m^{*2}\right)\right) \quad (4.5)$$

where an integration constant is chosen according to equation (4.1a) to be $H_0 = \frac{\kappa}{g_o}\sqrt{\frac{\Lambda_0}{3}}$ and a constant C is the same as in (4.4). Further one can check that equation (4.1a) is satisfied up to $\delta\phi^4$ terms which are beyond of our approximation. For the succeeding analysis it is convenient to represent ϕ and H as follows

$$\phi = 1 - \bar{C}e^{-\bar{r}t}\sin(\bar{\nu}t+\bar{\varphi}) \quad \text{and} \quad H = \frac{\kappa}{g_o}\sqrt{\frac{\Lambda_0}{3}} - \frac{\kappa^2}{g_o^2}C_He^{-2\bar{r}t}\sin(2\bar{\nu}t+\varphi_H) \qquad (4.6)$$

where $\bar{C}$ and $\bar{\varphi}$ are arbitrary (integration) constants,

$$C_H = \frac{\bar{C}^2}{32}\sqrt{\bar{m}\bar{m}^*}\sqrt{(1+4\xi^2)^2+\xi^4m^2m^{*2}-\xi^2(1+4\xi^2)(m^2+m^{*2})},$$
$$\text{and } \varphi_H = 2\bar{\varphi}+\frac{\pi}{2}+\arctan\left(i\frac{\bar{m}-\bar{m}^*}{\bar{m}+\bar{m}^*}\right) - \arctan\left(i\frac{\xi^2(m^2-m^{*2})}{2+8\xi^2-\xi^2(m^2+m^{*2})}\right).$$

In a special case $\xi^2=0$ which corresponds to a p-adic string in the FRW Universe the story is much simpler because eigenvalues $\bar{m}_{\pm k}$ can be expressed in elementary functions since $m_k^2 = 4(\log 3 + 2\pi ki)$. Moreover, $\bar{m}_{\pm 0}$ are pure real and $\bar{m}_{+0}$ corresponds to a smooth approach of the tachyon field ϕ to the asymptotic value $\phi_0 = 1$ without oscillations. Expressions for $\delta\phi$ and H in this case are as follows

$$\delta\phi = \bar{C}e^{-\bar{m}t} \text{ and } H = \frac{\kappa}{g_o}\sqrt{\frac{\Lambda_0}{3}} - \frac{\kappa^2}{g_o^2}C_He^{-2\bar{m}t} \qquad (4.7)$$

where $\bar{m} = \frac{3}{2}H_0 + \sqrt{\frac{9H_0^2}{4}+4\log 3}$, $C_H = \frac{\bar{C}^2\bar{m}}{16}$ and H_0 is the same as in (4.5). Again one can check that equation (4.1a) is satisfied up to $\delta\phi^4$ terms.

Plots representing the tachyon field evolution both for $\xi^2 \approx 0.9556$ and $\xi^2 = 0$ are shown in figure 3.

5. Cosmological consequences and further directions

The obtained asymptotic solutions for the tachyon and Hubble parameter (4.6) lead to a number of interesting cosmological properties.

First, we mention that H tells us about the time point t_0 after which the solution is reasonable. Indeed, for H to be strictly positive we have to require $\sqrt{\frac{\Lambda_0}{3}} > \frac{\kappa}{g_o}C_He^{-2\bar{r}t}$.

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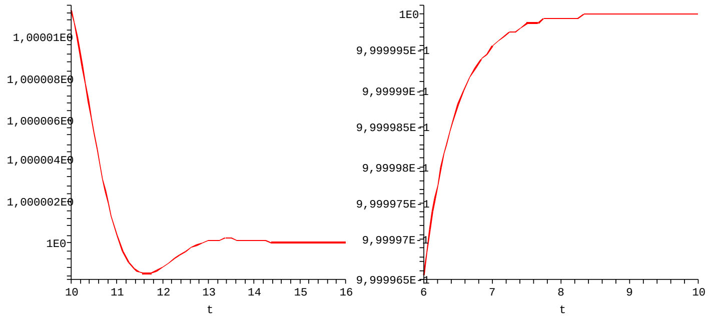

Figure 3: Tachyon field for $\xi^2 \approx 0.9556$ (left) and $\xi^2 = 0$ (right) for $\bar{C} = 1$, $\bar{\varphi} = 0$ and $\Lambda_0 = 3\cdot 10^{-4}$.

Solving this inequality w.r.t. the time we have $t > t_0 = -\frac{1}{2\bar{r}} \log\left(\frac{g_o}{\kappa C_H}\sqrt{\frac{\Lambda_0}{3}}\right)$. This is the characteristic time in question.

Then one sees that during the evolution of the tachyon field the Hubble parameter goes to a constant at large time. $\dot{H}$ obviously vanishes and moreover, both H and $\dot{H}$ do oscillate. The state parameter w also has an oscillating time behavior and goes asymptotically to -1. An explicit expression for it is of the form

$$w = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2}{3}C_H e^{-2\bar{r}t}\frac{-2\bar{r}\sin(2\bar{\nu}t + \varphi_H) + 2\bar{\nu}\cos(2\bar{\nu}t + \varphi_H)}{\left(\sqrt{\frac{\Lambda_0}{3}} - \frac{\kappa}{g_o}C_H e^{-2\bar{r}t}\sin(2\bar{\nu}t + \varphi_H)\right)^2}. \tag{5.1}$$

It is very interesting that w crosses the phantom divide $w = -1$ during the evolution. Such a crossing is forbidden in single field cosmological models with a local action. In our model we see that a non-locality breaks this restriction. The deceleration parameter q behaves very similar to w because its expression through H is very close to w

$$q = -1 - \frac{\dot{H}}{H^2} = -1 + C_H e^{-2\bar{r}t}\frac{-2\bar{r}\sin(2\bar{\nu}t + \varphi_H) + 2\bar{\nu}\cos(2\bar{\nu}t + \varphi_H)}{\left(\sqrt{\frac{\Lambda_0}{3}} - \frac{\kappa}{g_o}C_H e^{-2\bar{r}t}\sin(2\bar{\nu}t + \varphi_H)\right)^2}. \tag{5.2}$$

Hence the Universe exhibits an acceleration but because of oscillations quintessence and phantom phases change one each other with the time.

The scale factor a is related to H as $H = \dot{a}/a$ and can be readily found to be

$$a = a_0 e^{\int H dt} = a_0 \exp\left(\frac{\kappa}{g_o}\sqrt{\frac{\Lambda_0}{3}}t + \frac{\kappa^2}{g_o^2}C_H e^{-2\bar{r}t}\frac{\bar{r}\sin(2\bar{\nu}t + \varphi_H) + \bar{\nu}\cos(2\bar{\nu}t + \varphi_H)}{\bar{r}^2 + \bar{\nu}^2}\right) \tag{5.3}$$

where a_0 is an integration constant. At large times it has a simple exponential approximation

$$a = a_0 e^{\frac{\kappa}{g_o}\sqrt{\frac{\Lambda_0}{3}}t}.$$

To plot H and w we specify H_0 (or Λ_0) to be so small that the $\bar{r}$ and $\bar{\nu}$ in (4.6) are $\bar{r}_{+0}$ and $\bar{\nu}_{+0}$ respectively. This corresponds to the middle plot in figure 2. The explained above behavior of H, w, q, and a is visualized in figure 4. We point out that in spite of the presence of only one scalar field we observe similarities in the behavior of cosmological

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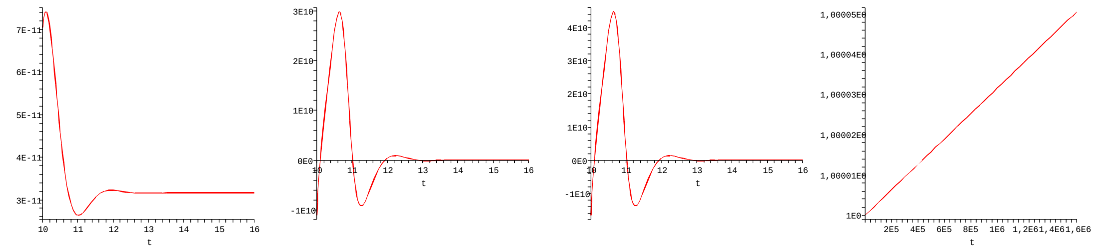

Figure 4: From left to right H, w, q, and a for $\kappa = g_o = \bar{C} = a_0 = 1$, $\bar{\varphi} = 0$ and $\Lambda_0 = 3 \cdot 10^{-21}$.

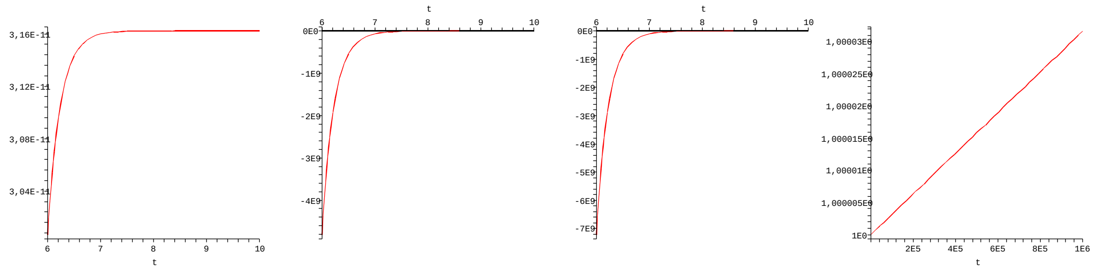

Figure 5: From left to right H, w, q, and a for $\kappa = g_o = \bar{C} = a_0 = 1$ and $\Lambda_0 = 3 \cdot 10^{-21}$ in the case $\xi^2 = 0$.

quantities with the two-field model analyzed in [34]. These are crossing of the phantom divide by w and the qualitative form of plots for H, w, and q. An order of magnitude of Λ_0 comes from an analysis performed in [44].

For $\xi^2 = 0$ one readily gets using (4.7) for the state and deceleration parameters

$$w = -1 - \frac{2}{3}\frac{2C_H \bar{m} e^{-2\bar{m}t}}{\left(\sqrt{\frac{\Lambda_0}{3}} - \frac{\kappa}{g_o} C_H e^{-2\bar{m}t}\right)^2}, \quad q = -1 - \frac{2C_H \bar{m} e^{-2\bar{m}t}}{\left(\sqrt{\frac{\Lambda_0}{3}} - \frac{\kappa}{g_o} C_H e^{-2\bar{m}t}\right)^2}. \tag{5.4}$$

We see that w is always less then -1 in this case and a crossing of the phantom divide does not occur. Thus the tachyon behaves purely like a phantom for $\xi^2 = 0$. The scale factor a in this case is found to be

$$a = a_0 e^{\int H dt} = a_0 \exp\left(\frac{\kappa}{g_o}\sqrt{\frac{\Lambda_0}{3}} t + \frac{\kappa^2}{g_o^2}\frac{C_H}{2\bar{m}} e^{-2\bar{m}t}\right) \overset{t\to\infty}{\approx} a_0 e^{\frac{\kappa}{g_o}\sqrt{\frac{\Lambda_0}{3}} t} \tag{5.5}$$

where a_0 is an integration constant. Plots for H, w, q, and a in the case $\xi^2 = 0$ are presented in figure 5. The behavior of the model in this case is very close to a single phantom field model solved in [44].

Remarkably, that both for $\xi^2 \approx 0.9556$ and $\xi^2 = 0$ the Big Rip singularity problem is avoided because w exhibits a non-trivial time dependence and consequently is not a constant less then -1.

The constructed solution reveals many properties expected from the cosmological point of view giving rise to an interest to a further investigation of this model as well as models coming from SFT in general. Apart from many other possible directions of developing this type of models we would like to mention ones which are seen as the most important.

It would be very interesting and at the same time very difficult to construct a solution to full non-linear equations (2.3). It seems that if possible it will be a numeric solution. However, even a numeric approach faces a difficulty of dealing with an exponential of the Beltrami-Laplace operator in a curved background. The main technical problem is that the Beltrami-Laplace operator itself contains an unknown function H to be determined while solving the equations.

A complete analysis of a stability of the obtained solution is also of great importance. A related issue which is an inclusion of other cosmic fluids and especially the Cold Dark Matter (CDM) forming about 1/3 of the present day Universe and an investigation of a dynamics of such a coupled model and its stability would be very important (see [45] as an example of a coupling of one phantom field to the CDM).

Also it would be interesting to find an exact solution to full equations with infinitely many derivatives in a curved background adding extra terms to the action. This is in accord with the analysis performed in [44, 46] where a small in terms of coupling constants correction to the potential makes the problem analytically solvable. An existence of an analytic solution provides a possibility of a qualitative analysis without a strong support of numeric methods.

Another important question is a consideration of closed string scalar fields, the closed string tachyon and dilaton, as well as their coupling to the open string tachyon. This problem is related to finding of lump solutions and a development in the flat background was started in [46]. Similar lump solutions have been constructed in [47]. An extension to the FRW Universe using a linearization of equations of motion developed in the present paper would be an interesting analysis of a role of closed string excitations.[8]

In spirit of a selected role of vector fields in a construction of the local phantom models without an UV pathology [24] it would be very interesting to incorporate a non-local SFT vector field in the study of the nonlocal rolling tachyon dynamics.

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A. A Solution to the characteristic equation

The equation in question is the following

$$\left(-\xi^2 m^2 + 1\right) e^{\frac{1}{4}m^2} = 3.$$

[8]See [48] for a discussion on a closed string tachyon and dilaton condensation.

It is a transcendental one and a solution to it can be represented analytically as

$$m_k^2 = \frac{1}{\xi^2} + 4W_k\left(-\frac{3}{4\xi^2}e^{-\frac{1}{4\xi^2}}\right) = 4\lambda + 4W_k\left(-3\lambda e^{-\lambda}\right)$$

where W_k is the k-s branch of Lambert W function satisfying a relation $W(x)e^{W(x)} = x$. It has infinitely many branches distinguished as W_k. A branch analytic at 0 is referred to as a principal one. There is exactly one such a branch and $k = 0$ is assigned to it. The branch cut dividing W_0 and $W_{\pm 1}$ is the part of the real axis $(-\infty, -1/e)$ and $-1/e$ is the branch point. The branch cut dividing all other neighbor branches is the negative real semi-axis and 0 is the branch point. For the principal branch $W_0(x)$ the image of the $(-\infty, -1/e)$ interval is $-\beta\cot\beta + i\beta$, where $\beta \in (0, \pi]$. For the branch $W_{-1}(x)$ the image of the $(-\infty, -1/e)$ interval is $-\beta\cot\beta + i\beta$, where $\beta \in (-\pi, 0]$. For all other branches $W_k(x)$ the image of the $(-\infty, 0)$ interval is $-\beta\cot\beta + i\beta$, where $\beta \in (2k\pi, (2k+1)\pi]$ if $k > 0$ and $\beta \in ((2k+1)\pi, (2k+2)\pi]$ if $k < -1$. For our purposes we are interested in the argument $x = -3\lambda e^{-\lambda} \approx -0.6042 < -1/e$, so the above properties are relevant. To understand the dependence $-\beta\cot\beta + i\beta$ we are going to consider the equation

$$ye^y = x$$

for $x < -1/e$. Assuming a complex solution $y = \alpha + i\beta$ we have

$$x = (\alpha + i\beta)e^{\alpha + i\beta} = \sqrt{\alpha^2 + \beta^2}e^{\alpha}e^{i\left(\arctan\frac{\beta}{\alpha} + \beta\right)}.$$

Since x is a negative real number we have to require a relation $\arctan\frac{\beta}{\alpha} + \beta = (2k+1)\pi$ which yields $\alpha = -\beta\cot\beta$ where a branch of the cotangent is the origin of branches of W function. Using this expression for α we can write $x = -\frac{\beta}{\sin\beta}e^{-\beta\cot\beta}$. Here the dependence on β is manifestly odd. From the point of view of W function the change $\beta \to -\beta$ is the change of branches $k \to -1-k$. This means the relations

$$\operatorname{Re} W_k(x) = \operatorname{Re} W_{-1-k}(x), \quad \operatorname{Im} W_k(x) = -\operatorname{Im} W_{-1-k}(x)$$

hold for $x < -1/e$. Moreover absolute values of real and imaginary parts of $W_k(x)$ grow when k grows for $k \geq 0$ or k decreases for $k \leq -1$. The latter is not evident and can be derived from the series expansion of W function.

For our value of ξ^2 all roots are complex with non-zero real and imaginary parts. Denoting $m_k^2 = \alpha_k + i\beta_k$ we have

$$m_k = r_k + i\nu_k, \quad r_k = \frac{|\beta_k|}{v}, \ \nu_k = \operatorname{sign}(\beta_k)\frac{1}{2}v, \quad v = \sqrt{-2\alpha_k + 2\sqrt{\alpha_k^2 + \beta_k^2}}$$

where for all square roots the arithmetic branch is chosen. In this case $a_k > 0$. Another root is $m_k = -(r_k + i\nu_k)$. Using the symmetry $\alpha_k = \alpha_{-1-k}$, $\beta_k = -\beta_{-1-k}$ we have for our definition of m_k that $m_k = m^*_{-1-k}$. Also for our value of ξ^2 all $\alpha_k < 0$.

Without an approximation $u = \tilde{u}$ the initial equation should be modified as follows

$$\left(-\xi^2 m^2 + 1\right)e^{\frac{1}{4}m^2} = 3 - 2\chi + 2\chi e^{-\frac{1}{4}m^2}.$$

A new parameter χ is used to interpolate between previous case ($\chi = 0$) and real situation ($\chi = 1$). This equation cannot be solved in terms of Lambert function. The easiest way to gain any information on roots is to make use of numeric methods. First, one can just plot the dependence on m^2 to visualize that there are no real roots. Thus we are going to represent as above $m^2 = \alpha + i\beta$ with α and β real and $\beta \neq 0$. Since unit is a positive real number we have to solve two equations

$$-\xi^2\alpha + 1 + 2\chi e^{-\frac{1}{2}\alpha} - \xi^2\beta\cot\frac{\beta}{4} = 0$$
$$\cos\frac{\beta}{4}e^{\frac{1}{4}\alpha}\left(-\xi^2\alpha + 1 - 2\chi e^{-\frac{1}{2}\alpha} + \xi^2\beta\tan\frac{\beta}{4}\right) = 3 - 2\chi$$

where the first one can be even solved analytically to yield

$$\alpha = 2W\left(\frac{2\chi}{\xi^2}e^{\frac{1}{2}\left(\beta\cot\frac{\beta}{4} - \frac{1}{\xi^2}\right)}\right) - \beta\cot\frac{\beta}{4} + \frac{1}{\xi^2}$$

In this expression we take 0 branch of the Lambert function as the only branch giving real values on the positive real semi-axis (the argument of W function is obviously real and positive). Further substitution into the second equation and numeric solution, say in Maple, brings out a sequence of roots for m^2 which has similar qualitative properties as already known in the first half of this appendix. Namely, there are no real roots, they form complex conjugate pairs, real and imaginary parts grow accordingly and any root with $\alpha > 0$ has a corresponding root with the real part equal $-\alpha$. Again branches of the cotangent develop branches in a solution for m^2. Of course, numeric values are changed compared to the previous simpler equation.

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